Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Fifth Edition.* Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Stewart Craven, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Stewart Craven at **numeratecitizen@mac.com**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

▲ Table of Contents

Up Front

Regular Features

Special Features

Articles

Notices

Advertisements

parallelogram whose area is equal to half that of the quadrilateral. This can be seen by cutting off the "ears," triangles *AKN, BLK, CML,* and *DNM*, and covering the parallelogram *KLMN*. Students might try to do this with a quadrilateral cut out of Bristol board.

2. Let *ABCD* be a quadrilateral and *K, L, M, N* be the midpoints defined in the previous problem. Join *KM* and *LN* and let them intersect at the point *O*. Then the quadrilateral is partitioned into four smaller quadrilaterals *AKON, BLOK, CMOL, DNOM*. The sum of the areas of two of these smaller quadrilaterals that do not have a common side is equal to the sum of the areas of the other two smaller quadrilaterals. In other words, the sum of the areas of the quadrilaterals *AKON* and *CMOL* is equal to the sum of the areas of the quadrilaterals *BLOK* and *DNOM*.

   (All the students need to know is that two triangles on equal bases and between the same heights have the same area. Encourage imagination and discourage computation and the use of formulae. One might hope that someone will think to join the vertices of the quadrilateral to *O*. Did this happen with your group?)

3. Let *A, B, C, D* be four distinct points on a page. There are six pairs of these points: *(A, B), (A, C), (A, D), (B, C), (B, D), (C, D)*, and we can measure the distances between each of these pairs. Normally, we will get six different numbers, but there are some configurations where there will be fewer different numbers. Find all the configurations in which you get only two different numbers.

   (This should be a question that is easy to get into. An example that most classes hit on right away is the set of four vertices of a square; this helps define the problem for those unclear about what is being asked. In all, there are six possibilities that are more or less difficult to find. It is a remarkable group that finds all six, but most will find two or three more.)

I look forward to hearing from you about these investigations and about others that you have done with your students. My e-mail address is **barbeau@math.utoronto.ca**. ▲

▲ Go Fly a Kite: Using Diagonals to Classify Quadrilaterals

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Classifying quadrilaterals and other polygons is an important task in the learning of geometry. This kind of activity requires that students not only know the parts and properties of quadrilateral figures, but that they also know how these parts and properties are interrelated within and among the figures. Pierre van Hiele (1986) identified the level at which this activity takes place as informal deduction (see Figure 1). At this level, students do not produce formal proofs and often are convinced by measuring and verifying. Activities at the level of informal deduction are necessary to assist students in the transition to the level of deduction, which is the level at which much high school geometry instruction takes place. According to van Hiele, the attainment of levels is not age-related, and thus, we need to do these sorts of activities with anyone who is at Level 2. This, then, includes our high school students. I have done the activity I present in this article with students from Grades 5–12 and with teachers.

Level 0 (Basic Level): Visualization; Students recognize figures as total entities (triangles, squares), but do not recognize properties of these figures (right angles in a square).

Level 1 (Analysis): Students analyze component parts of the figures (opposite angles of parallelograms are congruent), but interrelationships between figures and properties cannot be explained.

Level 2 (Informal Deduction): Students can establish interrelationships of properties within figures (in a quadrilateral, opposite sides being parallel necessitates opposite angles being congruent) and among figures (a square is a rectangle because it has all the properties of a rectangle). Informal proofs can be followed, but students do not see how the logical order could be altered, nor do they see how to construct a proof starting from different or unfamiliar premises.

Level 3 (Deduction): At this level, the significance of deduction as a way of establishing geometric theory within an axiom system is understood. The interrelationship and role of undefined terms, axioms, definitions, theorems, and formal proof is seen. The possibility of developing a proof in more than one way is seen.

Level 4 (Rigor): Students at this level can compare different axiom systems (non-Euclidean geometry can be studied). Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

Figure 1. Van Hiele: Levels of Geometric Thought. From **euler.slu.edu/teachmaterial/Van_Hiele_Model_of_Geometr.html**

I was inspired to create this activity by my experiences as a child, when I would construct a kite from two sticks, twine, and paper. To construct a kite, I first made sure that I had a longer stick and a shorter stick to form the spars. I used a piece of twine to determine the middle of the shorter stick by matching the length of the stick with twine, folding it in half, and using that piece to mark the middle. Then, using twine, I would tie the shorter stick onto the longer stick so that the middle of the shorter stick was above the middle of the longer stick and the two sticks were perpendicular to each other. Next, I would take twine and connect it to each end of each stick so that a quadrilateral figure (kite) was made. I would then cover this area with paper, and glue flaps over the twine perimeter.

Asking the Inverse Question

In many cases, especially at lower van Hiele levels, we have students work from the figures to the parts and properties. To assist students in the development of their mathematical reasoning, it is important that they be asked to invert this approach (Freudenthal, 1978). In this article, I propose an activity that addresses the classification of quadrilaterals by reasoning from the properties of the diagonals of quadrilaterals to the shape that belongs to them. So, instead of working toward the idea that a square is a figure in which the diagonals are equal and are perpendicular bisectors, we invert the task as follows:

When you think of four-sided figures (quadrilaterals), you might think of a kite. A kite is actually a special kind of quadrilateral. As you know, the skeleton of a traditional-looking kite is made with two intersecting sticks (spars) and some twine going from end to end. The two sticks (spars) are called the diagonals of the quadrilateral.

Find a systematic way of determining the maximum number of different quadrilateral shapes you can design this way.

Remember, all you have is "sticks and twine." Decide on what sorts of things you can change so that the design is different from any one you have designed up to this point. Report on your method of investigation, and evaluate how sure you are that you have found every possible different quadrilateral. If there is something special about a given quadrilateral design, then give it a name. Justify your choice of name, or demonstrate that the name that you have chosen is appropriate.

Materials

- Plastic straws (flat coffee stirrers are preferred) or flat dry pasta, such as linguini (if allowed)
- Scissors
- Rulers
- Protractors
- Geometric software, such as The Geometer's Sketchpad

Discussion

The key to this task is the emphasis on a systematic approach to finding all possibilities. How will a student know that all possible scenarios have been determined? The following is a way to scaffold this for students who may need assistance in exploring the possibilities systematically. (I have found that many are not inclined to do this without some prompting.)

Even though students may think that a certain quadrilateral has been created, they must verify or prove their conjecture. Allow students to do this at van Hiele Level 2, but promote Level 3 by posing probing questions. Using concrete materials or computer software, such as The Geometer's Sketchpad, allows almost all students to enter this problem in a way that encourages exploration. Dry spaghetti (if allowed) or coffee stir straws are very useful, since these can be easily broken or cut into different sizes. These materials can represent the spars (diagonals), as well as the twine

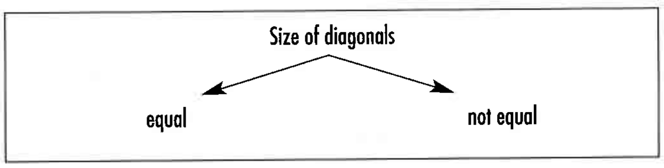
(sides). Equality of measures can be done by comparison or by using a ruler. The use of software such as The Geometer's Sketchpad may help students (at van Hiele Level 2) move from concrete reality to a medium in which they can design, measure, label, and verify. At a more advanced van Hiele level (3), students should prove their findings. I have provided one example proof below.

Scaffolding a Systematic Approach to Quadrilateral Classification

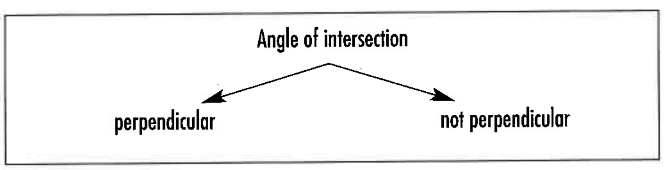
There are fundamentally three attributes that can be varied for diagonals:

1. their relative size
2. the angle of intersection
3. how they intersect relative to each other

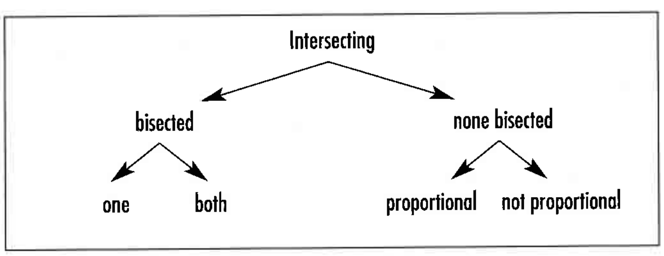
For relative size, we can say that the two diagonals are either equal or they are not:



For the angle of intersection, it may be helpful to notice that it must be between 0 and 180 degrees and that a clever division may be to say that it is either 90 degrees or it is not:



How two diagonals can intersect proves to be the more complicated issue. This is represented as follows:



This schema also provides a very nice hierarchical classification system in which all three elements come together as a conceptual tree diagram (See Figure 2).

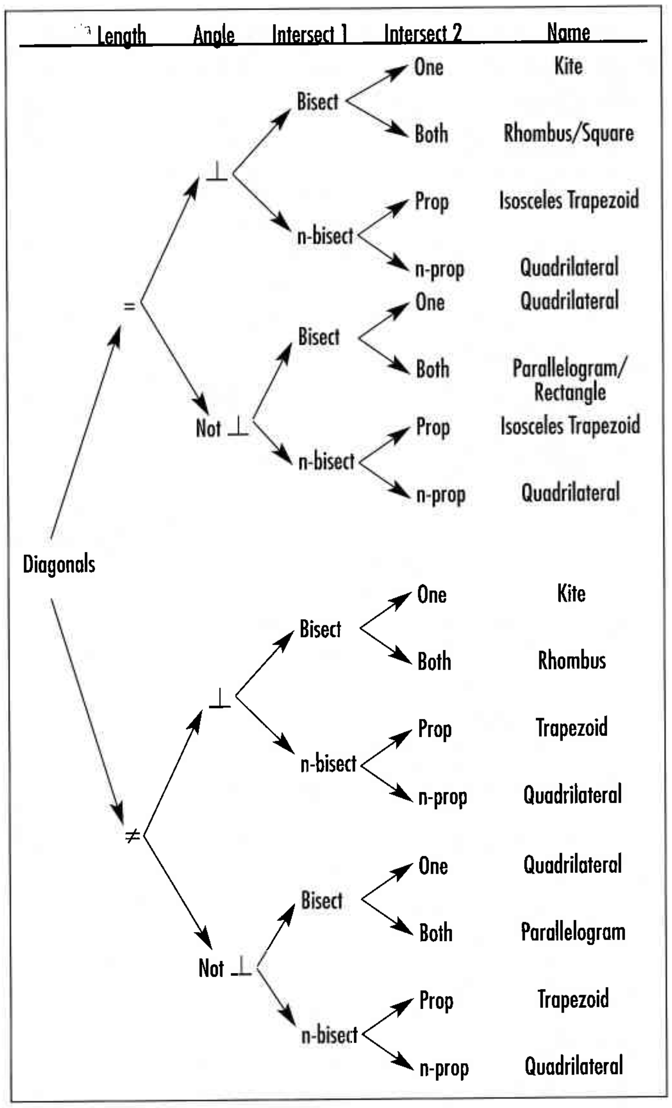


Figure 2. A systematic approach to quadrilateral classification.

The system in Figure 2 is represented pictorially in Figures 3, 4, 5, and 6.

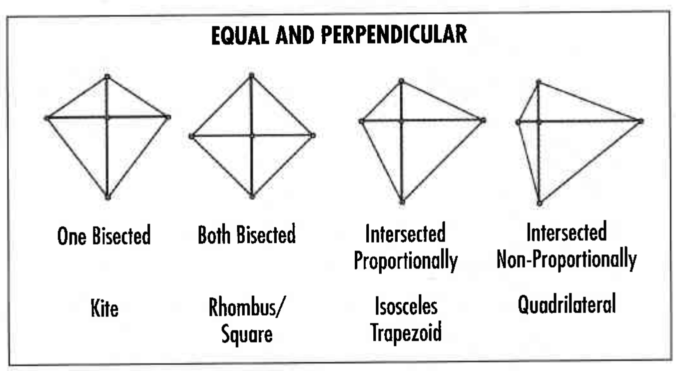


Figure 3. Equal and perpendicular diagonals.

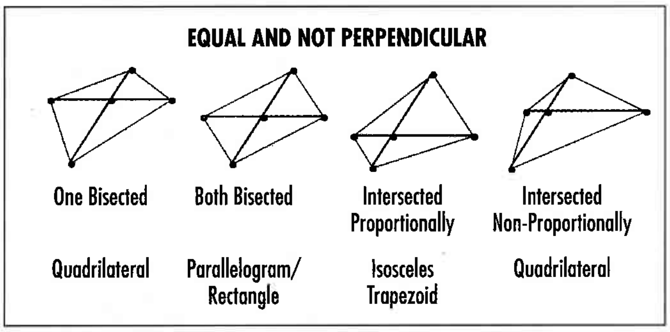


Figure 4. Equal and not perpendicular diagonals.

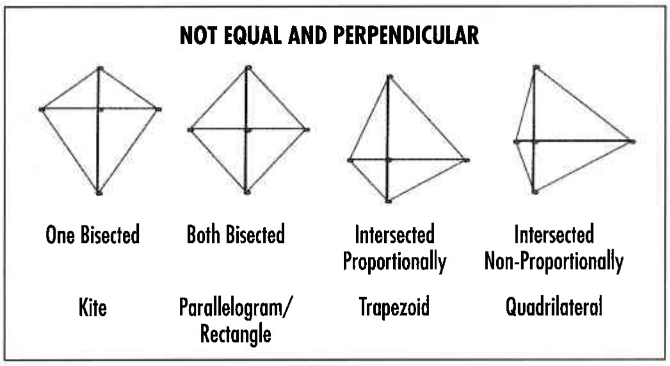


Figure 5. Not equal and perpendicular diagonals.

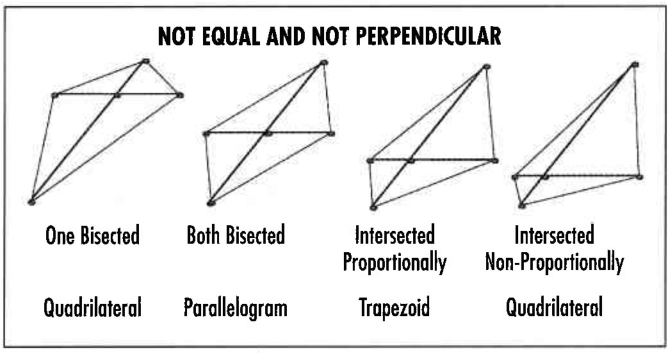


Figure 6. Not equal and not perpendicular diagonals.

The Case of Two Diagonals Intersecting Proportionally

In my experience, the case of two diagonals intersecting proportionally is rarely obvious to students. During the exploration, some prompting from the teacher is required. Even in doing this activity with teachers, I find that this is not an immediate thought. It is the trapezoid that makes this activity all the more interesting and rich. For example, after this activity, one can think of a rectangle as a trapezoid. To show that a quadrilateral is a trapezoid, we need to establish that one pair of opposite sides is parallel. Let's explore this further.

In order to demonstrate that two opposite sides are parallel, we consider one of the two diagonals to be a transversal of the pair of opposite sides we wish to show parallel. In the given situation, we cannot directly deduce similarity from the proportionality of the corresponding diagonal segments (see Figure 7). We will see that two similar triangles are formed, with these corresponding parallel sides as bases. In other cases, where the proportionality factor equals one, two congruent triangles are formed with parallel pairs of opposite sides, and we get results such as rectangles, squares, and parallelograms (see Figures 3, 4, 5, and 6). When neither diagonal bisects and is not segmented in proportion, we do not obtain a special quadrilateral, just a generic one.

We will now consider a deductive proof that the quadrilateral will have to be a trapezoid when the two diagonals intersect proportionally at van Hiele Level 3, using the law of cosines and sines.

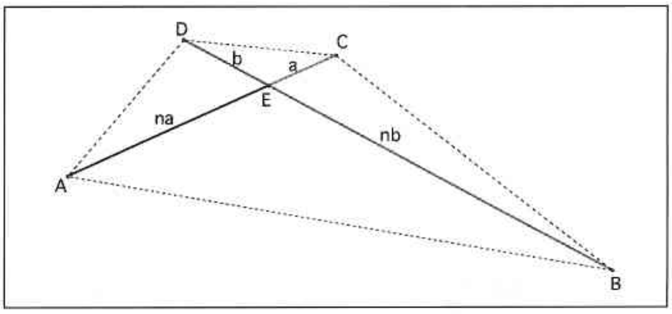


Figure 7. Diagonals intersecting proportionally.

In Figure 7, the diagonals $\overline{AC}$ and $\overline{BL}$ intersect proportionally in E. To show that the figure is a trapezoid, we need to show that $\overline{DC}$ and $\overline{AB}$ are parallel. This, in turn, can be shown by proving that either $\angle EAB \cong \angle ECD$ or by showing $\angle EBA \cong \angle EDC$. This then will show that $\overleftrightarrow{AC}$ or $\overleftrightarrow{DB}$ is a transversal of $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ with congruent alternate interior angles, thus showing that $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ are parallel.

Step 1

Use the law of cosines to show that $\overleftrightarrow{DC}$ *and* $\overleftrightarrow{AB}$ are proportional by the same factor n.

$$DC^2 = a^2 + b^2 - 2ab\cos(\angle CED)$$
$$AB^2 = (na)^2 + (nb)^2 - 2(na)(nb)\cos(\angle BEA)$$
$$= n^2\left[a^2 + b^2 - 2ab\cos(\angle BEA)\right]$$

Because $\angle CED$ *and* $\angle BEA$ are vertical angles, they are congruent. From this follows that: $AB^2 = n^2 DC^2$ and thus that $AB = nDC$.

Step 2

Use the law of sines in both triangles to show that $\angle EAB \cong \angle ECD$.

$$(1)\quad \frac{\sin(\angle CED)}{DC} = \frac{\sin(\angle ECD)}{b}$$

$$\Rightarrow \sin(\angle CED) = \frac{DC}{b}\sin(\angle ECD)$$

$$(2)\quad \frac{\sin(\angle BEA)}{nDC} = \frac{\sin(\angle EAB)}{nb}$$

$$\Rightarrow \sin(\angle BEA) = \frac{nDC}{nb}\sin(\angle EAB)$$

$$= \frac{DC}{b}\sin(\angle EAB)$$

From (1) and (2) follows that $\angle ECD = \angle EAB$.

QED

Alternatively, we can reason as follows in Step 2: Because all three pairs of corresponding sides of the two triangles are in the same proportion, the two triangles must be similar. Similar triangles have congruent corresponding angles. In essence, the law of sines represents this relationship.

I hope that you and your students will gain much pleasure from this activity. In this case, posing the inverse question creates a more challenging environment. Students at van Hiele Level 2 are sometimes easily discouraged when the tasks given to them are at Level 3. According to van Hiele and others, no matter what the age or grade level of the students, it is necessary to deliberately design activities that will help them transition through these levels. This does not appear to happen "naturally."

I have found that this activity assists with the Level 2 to Level 3 transition.

References

Freudenthal, H. (1978). *Weeding and sowing: Preface to a science of mathematical education*. Dordrecht, Netherlands: D. Reidel Publishing Company.

Van Hiele, P.M. (1986). *Structure and Insight*. Orlando, FL: Academic Press. ▲

▲ Directors' Dialogue

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OAME Board of Directors Meeting February 6–7, 2009

Friday, February 6, 2009

The Board of Directors meeting began with remarks by Anna Jupp, OAME's president. She reflected on change and how it influences our practice as teachers. To illustrate this fact, she showed the video "Shift Happens," available at **www.youtube.com**. Afterward, a member of the Constitution Committee directed members to consider changes to OAME's Board. Members discussed the appropriate size of the Executive and Board of Directors, whether members should be locally or provincially elected, and whether there should be a balance of elementary and secondary educators on the Board.

Saturday, February 7, 2009

The Equity Committee met over breakfast. In planning for the OAME annual conference, Sue Melville reported that Kelly-Lee Assinewe of the N'Swakamok Friendship Centre Alternative School in Sudbury will speak about alternative approaches to educating First Nations students during the Equity breakfast. She also announced the members that would be featured during the panel discussion: Shaun Knowles, Barb Guglielmi, and Kelly-Lee Assinewe. The Equity Committee devised a list of questions for the three guests. Future plans for the committee include a study of Marion Small's new book on questioning and differentiated instruction. Other ideas include the development of differentiated assessments and a sample grade book.

OAME executive director Dave Hessey began a presentation of OAME's finances. Dave told members that OAME expenses are currently below budget; Connie Quadrini, one of the annual conference co-chairs from 2008, shared that last year's OAME conference yielded a profit. Executive director Sue Hessey announced that some issues of the *Abacus*, the elementary math